Beyond belief

The great warrior rubbed more oil on his already glistening skin and made some perfunctory warm-up movements. Achilles wasn't worried: what an absurd idea to have him race against a tortoise, even though the animal had a fifty meter start. A few strides, and he would be ahead.

But he hadn't reckoned with this thought: in order to overtake the tortoise, he first needed to get to where the tortoise started. While he was doing that, the tortoise would have made some forward progress, so before Achilles could overtake he would need to get to the tortoise's new position. But in the time it took Achilles to do that, the tortoise would have made further progress. Achilles would still be behind. He was never going to catch the tortoise. His early confidence was misplaced.

We all know that Achilles will win, but the argument in the previous paragraph purports to prove that he cannot win. We have a *paradox*, a word whose Greek roots suggest something beyond belief: we cannot believe both that Achilles will win and that he will not, for that would be a contradiction.

This paradox of *Achilles and the tortoise* was described by Aristotle and attributed to the philosopher Zeno of Elea, who lived some 400 years before the Christian Era. As with many paradoxes, we have an apparently compelling argument for an apparently absurd conclusion. To restore equilibrium, we either have to accept the conclusion or reject the argument. In the one case, we have to say that however much the conclusion seemed absurd at first sight, reflection shows that it is not really absurd, but indeed correct. In the other case, we need to show what went wrong with the apparently compelling reasoning.

Are paradoxes just trivia, good for passing a dull moment, but of no serious significance? Some of them are certainly trivial, like the one about the barber who shaves all and only those who do not shave themselves. It seems to follow that the barber both does and does not shave himself. For if he does not shave himself, and he shaves *all* those who do not shave themselves, then he shaves himself. But if he shaves himself, and he shaves *only* those who do not shave themselves, then he does not shave himself. We know how to deal with this one: there can of course be no such barber.

Although the *barber paradox* seems trivial, it looks quite similar to a paradox that has had a huge impact on mathematical logic, *Russell's paradox.* The name honors Bertrand Russell, one of the most famous philosopher-mathematicians of the twentieth century. He was reflecting on the work of the German mathematician Georg Cantor, generally regarded as the inventor of set theory. Cantor used the German word "Menge" for what would now be called a set, and Russell uses "class" in much the same way. It is natural to suppose that any things can be gathered together to form the members of a class. For example, the class of horses has as members just horses, and nothing else. The class of horses is not a member of the class of horses, for it is a class and not a horse. By contrast, the class of classes with more than two members is a member of itself, since it has more than two members. Russell argued that the very notion of a class leads to a contradiction. Consider the "Russell class": the class of just those classes that are not members of themselves. We have barber-like reasoning: if the Russell class is a member of itself, and has as members *only* classes that are not members of itself, then the Russell class is not a member of itself. But if the Russell class is not a member of itself, and has as members *all* those classes that are not members of themselves, then it is a member of itself. The Russell class is a member of itself and is not; which is impossible. This is *Russell's paradox*.

Isn't it just as trivial as the barber paradox? All we have to say is that there is no such class as the Russell class, just as we said there was no such barber. But this is not the easy fix it seems to be, since we are also inclined to hold that any *condition*, like the condition *being a horse*, or the condition *being a class*, determines a class. The horse-condition determines the class of horses, the class-condition determines the class of classes. The condition need not be consistent: the condition *being a round square* determines the class of round squares. There exists such a class, though it is one with no members. Hence we do not have to restrict our conditions to consistent ones: any condition at all, it seems, must determine a class. In that case, the condition *being a non-self-membered class* determines a class, from which it follows that the Russell class exists. We have a new paradox, or an extension of the original one: there cannot be such a class as the Russell class, because to suppose there is leads to contradiction, but there must be such a class as the Russell class, since it is determined by a corresponding condition. Much of the huge and technical book *Principia Mathematica*, which Russell wrote with his colleague Alfred North Whitehead, and which was published in three volumes between 1910 and 1913, is concerned with this issue and its ramifications. These volumes are dense: they cannot be recommended as beach reading.

Paradoxes can crop up in any subject area, for example in morality. Consider the choice that Sophie had to make in Willian Styron's *Sophie's Choice*. The guard in the concentration camp told Sophie she had to select one of her two children to be killed. If she refused, they would both be killed. She ought to do her best for her children, so she should do whatever it takes to save at least one. But the only way she can achieve that is to do something appalling: select a child to be killed. This is an agonizing example of a conflict of obligations: Sophie ought not to cause one of her children's death; but also she ought not to refuse to prevent one of her children's death. The problem is that she cannot act in accordance with both these obligations. Does this show that morality is contradictory, and so "beyond belief"? If so, does this mean that it cannot be trusted?

Paradoxes come in degrees. Easy paradoxes, like the barber, are easy to dispose of. Hard paradoxes, like Russell's paradox, make it hard to find a coherent position: one seems obliged to accept something beyond belief. *Sophie's paradox* is intermediate, and there are different views. I think the right thing to say is that there are genuinely conflicting moral obligations, but conflicting obligations are not contradictory: both can obtain, even if one cannot act in accordance with both. It would be contradictory to suppose that it is both the case and also not the case that one ought to do something. But there is nothing contradictory about there being an obligation to do something and also an obligation not to do that thing. Compare: it would be contradictory to suppose that there are animals who do and do not eat grass, but there is nothing contradictory about there being animals who eat grass and animals who do not. They have “conflicting diets”.

Paradoxes are a mixed bunch, and there are not many general truths about them. I’ll mention a couple of further ones that are hotly discussed in current philosophy.

Given a heap of sand, removing a single grain will not destroy it: you will still have a heap. But, taken generally, this is paradoxical: if taking away a grain from a heap always leaves a heap, you could not destroy a heap by taking away the grains one-by-one. Yet, obviously, you could destroy a heap that way. Paradoxes like this are called *sorites paradoxes*. They go back thousands of years, and arise from the vagueness of words like “heap” or perhaps from the nature of heaps: we know of no number which is the smallest possible number of grains with which we can make a heap.

Vagueness is omnipresent in our lives and thought. When does a child become an adult? It’s tempting to say that if someone is a child at a time, they are still a child a second later. But this, like the paradox of the heap, suggests that however many seconds pass, a child will never grow into an adult; and this is beyond belief. Some people think that this kind of vagueness is important to the debate over abortion: if a child has full human rights at birth, as almost everyone believes, then it had full human rights a second earlier, and we can continue the argument back to the moment of conception, with “full human rights” preserved along the way. Colors are another familiar example of vagueness, and they give rise to the same kind of paradox. Imagine a series of patches of color starting with a red patch. The patches become slightly less red, moving through orange to yellow. Every adjacent pair of patches is very similar, but they are definitely not all red. It seems we have to draw the line somewhere, saying that there is a last red patch, with an adjacent patch that is not red, despite being extremely similar in color. Walking down a mountain, what is the last step you took when you were still on the mountain, the next step being your first step not on it? Vagueness is everywhere.

Philosophers have gone through all kinds of contortions to address sorites paradoxes. Some say that when we attend properly to the context in which vague words are used the paradox will go away. Some say that we should not think in terms of absolute truth and falsehood, but only of degrees of truth, degrees which may gradually diminish as we get further and further from clear cases. One of the boldest ideas is called epistemicism, associated especially with Oxford philosopher Timothy Williamson: it’s the view that there really are sharp boundaries, it’s just that we can’t know where they lie. The slogan is “vagueness is ignorance”. There is a smallest number of grains which can be made into a heap, but we can’t know what that number is. There is a moment at which a child becomes an adult, but we can’t know when that magic moment occurs. Borderline cases are ones we can’t know how to classify, despite the fact that there is a classificatory fact of the matter. The problem for the epistemicist is to explain how we can be in principle ignorant of facts about borderlines. If the sharp boundaries are really there, why can’t we find them?

One of the hardest paradoxes is called the *Liar Paradox*. It's exemplified by a shirt we used to sell in the University of Texas philosophy department. On the front was printed "What's on the back of this shirt is true". On the back was printed "What's on the front of this shirt is not true". Are the sentences true or not? Let's start with the one on the front, and suppose it is true. Then it truly says that what's on the back is true. But the sentence on the back says that the one on the front is not true. So if it's true, it's not true. Similar reasoning will show that if it's not true, then it is true.

We can simplify by considering a sentence that says of itself that it's not true, for example the sentence "This very sentence is not true". If it says this of itself truly, then it's not true. But if it's not true, then it is true, for *not true* is just what it says it is. It won't help to say that the sentence is meaningless or somehow defective, for a meaningless or defective sentence is not true – which is just what the sentence says it is.

After hundreds, perhaps thousands of years of debate, including very sophisticated technical debates in the past 100 years, there is still no agreed way of dealing with Liar paradoxes. I close by describing one radical suggestion for dealing with this and perhaps other very hard paradoxes.

The suggestion is that some sentences are both true and false, so that what seemed absurd about the conclusion of the Liar paradox is in fact correct. This is called "dialetheism". Its most salient exponent is the philosopher Graham Priest, based in philosophy departments in Australia (and also, more recently, in the USA). At first, dialetheism might not sound startling: we are used to saying that there is some truth in what someone said, but that it's not wholly true. For example, there's some truth in the claim that people are disposed to be generous, but it's not wholly true: there are exceptions. This makes it sound as if we regard some sentences as both true and false. But we do not do so in the radical way dialetheism demands. For what we normally suppose is that the true parts can be separated out from the false ones. Most people are generous: that (let's suppose) is entirely true. And it's entirely true that some people are not. So the generalization that people are generous is partly true and partly not true. Dialetheism's radical idea is that there are sentences in which truth and falsehood are inextricably fused: there's no separating out a false element from a true element.

Now we have a response to the Liar paradox: the problematic sentence is true, and also false; and so both true and also not true, just as the paradoxical arguments suggest. This response is not recommended for all paradoxes. It would be silly to suggest that Achilles both will and will not overtake the tortoise, or that the barber both does and does not shave all and only those who do not shave themselves. You can think of dialetheism as the ultimate solution, to be considered only when all else fails.

Is Achilles still trying to catch the tortoise? One standard response to this paradox is that the phrasing misleads us into thinking that Achilles has an infinite number of runs to run: first to run to the tortoise's starting position, then to run to the position the tortoise gets to while Achilles is doing his first run, and so on. We see that these runs can be arranged in a series that does not terminate. And then we think: poor Achilles! Infinitely many runs to run in a finite time! Not possible!

But this is not the only way to think of motion. Consider moving your hand a couple of inches across your desk. If space is infinitely divisible, you have moved through infinitely many regions in a finite time. Sounds impossible? We all know that it's not impossible, and hence that there is a perfectly non-paradoxical description of what is happening. This is what needs to be transferred to the race with Achilles, possibly enhanced by the distinction between the limit of an infinite convergent series and a member of the series. Though while we're working on that, beware of the slow but indefatigable tortoise!