

## Spotty scope

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### 1. Branched quantifiers

Henkin introduced what he called ‘partially ordered quantifiers’, now often called ‘branched/branching quantifiers’ (Quine 1969), and claimed that adding them to an otherwise first-order language increased its expressive power.<sup>1</sup> In a first-order language, truth conditions can be affected by which existential quantifiers are placed in the scope of which universal quantifiers. An interpretation of an existential within the scope of a universal quantifier depends upon the interpretation of the universal quantifier. Sometimes this dependence is unwanted. To use a standard example, one might wish so to quantify a four-place open formula  $Rwxyz$  with two  $\forall\exists$  pairs,  $\forall w\exists x$  and  $\forall y\exists z$ , yet in such a way that neither existential quantifier depends upon the universal quantifier outside its pair. This desire is unattainable in first-order logic. But it can be expressed in the branching mode:

$$\left. \begin{array}{l} \forall w\exists x \\ \forall y\exists z \end{array} \right\} Rwxzy$$

The co-ordinate positions of the two quantifier pairs represents the intended mutual independence.

Quite which, if any, English sentences should give rise to the desire in question is more controversial. Hintikka (1976) suggested the following:

Some relative of each villager and some relative of each townsman hate each other.

Intuitively, we understand this to have a structure on which it is true iff the result of interchanging ‘villager’ and ‘townsman’ is true. Reading  $Rwxzy$  as abbreviating ‘ $w$  is a villager and  $x$  is a relative of  $w$  and  $y$  is a townsman and  $z$  is a relative of  $y$  and  $x$  and  $z$  hate each other’, the effect of the interchange would be achieved in a first-order framework by switching the order of the  $\forall\exists$  quantifier pairs; but in this framework the switch

<sup>1</sup> The standardly cited source is Henkin 1961. For philosophers, Hintikka 1974, 1976 and Barwise 1979 are seminal. For an overview which focuses on the logical aspects, with a bibliography of nearly 80 works, see Krynicky and Mostowski 1995.

would not preserve equivalence:  $\forall w\exists x\forall y\exists zA$  and  $\forall y\exists z\forall w\exists xA$ , regarded as first-order sentences, are not in general equivalent. This encourages the thought that the displayed English sentence is not adequately formalizable in a first-order language, though the problem is clearly one which will not affect a language with branched quantifiers: in such a language, the quantifier pairs do not have a relative order. The present paper remains neutral on the question which English sentences, if any, require to be interpreted by branched quantification. The point of mentioning this form of quantification is to make a general point about scope.

The branching formula displayed above has the following feature:  $Rwxyz$  is in the scope of both quantifier pairs, but neither of these pairs is in the scope of the other. This is a feature of scope which is impossible in many logics (including first-order logic), which are governed by the general rule that if something is in the scope of both  $x$  and  $y$ , then either  $x$  is in the scope of  $y$  or  $y$  is in the scope of  $x$ . When this principle fails, I say we have ‘spotty scope’.

One could represent scope relations by lines, in which case spotty scope will show up as broken lines, as in the following version of the branched formula displayed above, in which the underlining marks the scope of the  $\forall w\exists x$  pair, and overlining the scope of the  $\forall y\exists z$  pair:

$$\forall w\exists x \underline{\forall y\exists z} \overline{Rwxyz}$$

What the study of branched quantifiers shows is that there is nothing wrong with spotty scope. Whatever the other disputes about these quantifiers, it has not been doubted that this feature of their scope does not as such constitute any ground for rejecting them. My aim in this paper is to show how spotty scope can help make other problems seem less daunting.

## 2. *Intentional identity*

Geach invites us to consider the following sentence (I keep to his numbering):

- (3) Hob thinks a witch has blighted Bob’s mare, and Nob wonders whether she (the same witch) killed Cob’s sow. (Geach 1967: 147)

The sentence raises two problems. One is how we should explain the possibility of what looks to be identity, even when there is no object to bear the identity relation (there are no witches). This problem, which in my view is to be resolved by a reference-conditions account of referring expressions, will not be addressed here. The other problem is one of syntax or logic (to adopt Geach’s terminology). His initial formulation is in terms of the fact that the occurrence of ‘she’ falls into neither of Quine’s supposedly exhaustive categories *opaque* or *transparent*:

if the indirect-speech clauses in (3) are construed opaquely, then each clause must stand on its own syntactically; this is graphically shown by Quine's way of enclosing such clauses in square brackets; and Quine forbids syntactical liaisons, like the binding of variables, to cross this barrier. But on the face of it we have in (3) a pronoun, 'she' or 'the same', bound to an antecedent, 'a witch', that lies outside the clause containing the pronoun; so ... the clauses in (3) cannot be construed opaquely. (Geach 1967: 147)

The clauses cannot be construed 'transparently' without incorrectly interpreting (3) as entailing the existence of witches. So if opaque and transparent exhaust the possibilities, we are deadlocked.

Pronouns in natural language have a disgraceful habit of behaving in ways that Quine forbids. Cross-sentence anaphora shows that the conception of scope used in classical logic will not serve us in describing natural language, unless it is substantially modified. Another Quinean prohibition is infringed when we quantify into propositional attitude contexts. But as Geach notes, these are not the only 'difficulty' which (3) raises. In his words:

... (3) raises an even worse difficulty: a pronoun in one indirect-speech clause is on the face of it bound to a quantified phrase in *another* such oblique context; the scope of the quantified phrase thus seems *both* to lie wholly within the earlier oblique context *and* to cover something in the later context. I cannot even sketch a structure of operators that would make good logical sense of this. ... (Geach 1967: 150)

Spotty scope to the rescue.<sup>2</sup> The formal notion of scope is not properly defined except relative to a formalized syntax, but we know how to apply an analogous informal notion to natural language, and Geach is happy with this procedure ('I propose that the familiar jargon of binding, scope, and so on should be extended from symbolic language to appropriate pieces of the vernacular' (1967: 148)). Using underlining to mark the scope of Hob's attitudes (the scope of 'Hob thinks') and overlining to mark the scope of Nob's (the scope of 'Nob wonders'), we have

(3') Hob thinks a witch has blighted Bob's mare, and Nob wonders  
whether she (the same witch) killed Cob's sow.

<sup>2</sup> Quine (1969: 112) thanks Geach for pointing out a (more or less English) sentence which is a candidate for formalization by branching quantifiers. Spotty scope may have been on the tip of Geach's tongue.

'She' is in the scope of Hob's thinking and in the scope of Nob's wondering without either of these latter being in the scope of the other; this is the mark of spottiness. To say this is not to provide the needed systematic semantics for such sentences, but the analogy with the demonstrably coherent spottiness of branched quantifiers should encourage us to take the appearance of spottiness in (3) at face value, rather than try to find some alternative 'analysis' or 'logical form'. (Geach considers, and rejects, some attempts of this kind at the end of his paper.)

### 3. *Other applications*

In reporting reactions to fictions, we sometimes need, as Wiggins memorably put it, to have one foot in the fiction and one foot in the real world (1995: 108). According to the play *Coriolanus*, the hero is courageous, brutal, and warmongering, yet many people, possibly including Tony Blair, admire him. Him? Must we accept that Coriolanus himself is among the things there are, available to stand as a term in the two-place relation of admiration? Not if occurrences of the name, and of pronouns anaphorically dependent upon it, are within the scope of an operator like 'According to the fiction'. Given spotty scope, we can elect to take the ontologically modest approach, construing the talk of the play, the character, and Tony Blair's admiration, thus:

According to the play *Coriolanus*, the hero is courageous, brutal, and warmongering, yet Tony Blair admires him.

Both 'the hero' and its dependent 'him' are in the scope of 'According to the play', and hence do not require referents in order for the whole sentence to be true. The metaphor of two feet (one in the fiction, the other in reality) transmutes into the less metaphorical notion of spotty scope.

Walton (1990: 215–21) gives a complicated account of what is happening when Stephen, pointing at Willem van de Velde's *Shore at Scheveningen*, utters the words 'That is a ship'. Walton does not wish to give an account which commits to there really being fictional entities, so a problem arises for him about the referent of 'that'. On his view, since 'that' has no referent, Stephen's words cannot express a proposition. Since Walton apparently assumes that no sentence-forming operator can deliver a proposition-expressing sentence if the sentence upon which it operates does not express a proposition, we cannot construe Stephen as having truly said that, according to van de Velde's picture, that is a ship. Hence, says Walton, we must regard Stephen not as referring to anything or asserting anything, but only as pretending to do these things. This means that when we challenge Stephen ('barge, not ship') or elaborate ('indeed,

and with a nanteen sail'), we too are merely pretending to say things; an implausible result.

On the other hand, Walton is quite happy to use non-referring anaphoric pronouns in suitable embeddings to state literal truths. This is one such:

These difficulties pushed us toward the idea that Stephen was merely pretending to refer to something by means of the demonstrative and to say of it that it is a ship. (Walton 1990: 391)

The two occurrences of 'it' are dependent on the indefinite 'something' which has no witness, so these pronouns have no referent. Yet the quoted sentence is Walton's fully serious, non-pretend, statement of the position to which he has been pushed. This suggests that he would be happy with an account which mirrors this arrangement, that is, which attains truth despite having anaphoric pronouns dependent upon unwitnessed indefinites. Spotty scope makes this possible.

We need to decide what it is for a picture to be a picture of a ship. I suggest that it is this: according to the picture, there is a ship. We can now report Stephen's remark, incorporating part of its context (as we normally have to do when reporting demonstrative utterances), and exploiting spotty scope:

According to the picture, there is a ship, and Stephen said that it was a ship.

Both the indefinite 'a ship' and its dependent pronoun 'it' are in the scope of 'according to the picture'. The component 'Stephen said that' can be regarded as stating something that Stephen really did, not something he merely pretended to do; it lies outside the scope of 'according to the picture' and of any other such operator, explicit or implicit. This seems to me an advantage over Walton's actual account, and it makes use of no semantic or ontological doctrines which Walton repudiates.

#### 4. Conclusion

The fact that spotty scope is found in a rigorous formal setting (branched quantifiers) should give us confidence in using its natural language analog in describing a variety of problematic natural language constructions.

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